Correspondence between EED and the pre-Maxwell equations in SHP

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In SHP, the position of a classical (non-quantum) event in spacetime is a function of an external chronological parameter τ

$$x(\tau) = (t(\tau), \mathbf{x}(\tau)) \tag{1}$$

where τ plays a role similar to that of time t in non-relativistic Newtonian mechanics. Just as classical Maxwell fields are functions of position $\mathbf{x}$ and time t, SHP fields and potentials are functions of spacetime position $x = (t, \mathbf{x})$ and τ.

SHP electrodynamics defines *five* potentials

$$a(x,\tau) = \left(a^0, \mathbf{a}, a^5\right) \tag{2}$$

that produce the field strengths

$$\mathbf{e} = -\frac{1}{c}\frac{\partial \mathbf{a}}{\partial t} - \nabla a^0 \tag{3}$$

$$\mathbf{b} = \nabla \times \mathbf{a} \tag{4}$$

$$\boldsymbol{\epsilon} = \sigma \frac{1}{c_5}\frac{\partial \mathbf{a}}{\partial \tau} + \nabla a^5 \tag{5}$$

$$\epsilon^0 = \sigma \frac{1}{c_5}\frac{\partial a^0}{\partial \tau} + \frac{1}{c}\frac{\partial a^5}{\partial t} \tag{6}$$

where $\sigma = \pm 1$ depending on larger symmetry considerations, and $c_5 < c$ is a speed so that $c_5\tau$ is analogous to ct. The 3-vector $\boldsymbol{\epsilon}$ and scalar ϵ^0 are new field strengths arising from the τ-dependence of the potentials and fifth potential a^5.

These field strengths are invariant under the gauge transformations

$$a^0(x,\tau) \longrightarrow a^0(x,\tau) - \frac{1}{c}\frac{\partial}{\partial t}\Lambda(x,\tau) \tag{7}$$

$$\mathbf{a}(x,\tau) \longrightarrow \mathbf{a}(x,\tau) + \nabla\Lambda(x,\tau) \tag{8}$$

$$a^5(x,\tau) \longrightarrow a^5(x,\tau) + \sigma\frac{1}{c_5}\frac{\partial}{\partial \tau}\Lambda(x,\tau) \tag{9}$$

and it is convenient to take

$$\frac{1}{c}\frac{\partial a^0}{\partial t} + \nabla \cdot \mathbf{a} + \frac{1}{c_5}\frac{\partial a^5}{\partial \tau} = 0 \tag{10}$$

as the Lorenz condition.

The SHP field equations (pre-Maxwell equations) are

$$
\begin{aligned}
&\nabla\cdot\mathbf{e}-\frac{1}{c_5}\frac{\partial\epsilon^0}{\partial\tau}=ej^0 &&\nabla\times\mathbf{e}+\frac{\partial\mathbf{b}}{\partial t}=0\\
&\nabla\times\mathbf{b}-\frac{1}{c}\frac{\partial\mathbf{e}}{\partial t}-\frac{1}{c_5}\frac{\partial\boldsymbol{\epsilon}}{\partial\tau}=e\mathbf{j} &&\nabla\cdot\mathbf{b}=0\\
&\nabla\cdot\boldsymbol{\epsilon}+\frac{\partial\epsilon^0}{\partial t}=ej^5 &&\nabla\times\boldsymbol{\epsilon}-\sigma\frac{1}{c_5}\frac{\partial\mathbf{b}}{\partial\tau}=0\\
&\nabla\epsilon^0+\frac{1}{c}\frac{\partial\boldsymbol{\epsilon}}{\partial t}+\sigma\frac{1}{c_5}\frac{\partial\mathbf{e}}{\partial\tau}=0
\end{aligned}
\tag{11}
$$

This system can be put into correspondence with extended electrodynamics (EED) in the following way. We define

$$
C=\frac{1}{c}\frac{\partial a^0(x,\tau)}{\partial t}+\nabla\cdot\mathbf{a}(x,\tau)
\tag{12}
$$

as in EED. Then,

$$
C=-\frac{1}{c_5}\frac{\partial a^5(x,\tau)}{\partial\tau}
\tag{13}
$$

follows from the Lorenz condition (10). Now let the 4-vector potential be τ-independent

$$
a^0(x,\tau) = A^0(x) \tag{14}
$$

$$
\mathbf{a}(x,\tau) = \mathbf{A}(x) \tag{15}
$$

so that the pre-Maxwell $\mathbf{e}$ and $\mathbf{b}$ fields behave like Maxwell $\mathbf{E}$ and $\mathbf{B}$ fields

$$
\mathbf{e}(x,\tau) = -\frac{1}{c}\frac{\partial\mathbf{a}(x,\tau)}{\partial t}-\nabla a^0(x,\tau)=-\frac{1}{c}\frac{\partial}{\partial t}\mathbf{A}(x)-\nabla A^0(x)=\mathbf{E}(x) \tag{16}
$$

$$
\mathbf{b}(x,\tau) = \nabla\times\mathbf{a}(x,\tau)=\nabla\times\mathbf{A}(x)=\mathbf{B}(x) \tag{17}
$$

as is also the case in EED. With these conditions the scalar field strength reduces to:

$$
\epsilon^0 = \sigma\frac{1}{c_5}\frac{\partial A^0}{\partial\tau}+\frac{1}{c}\frac{\partial a^5}{\partial t}\ \rightarrow\ \frac{1}{c}\frac{\partial a^5}{\partial t} \tag{18}
$$

$$
\frac{\partial\epsilon^0}{\partial\tau} = \frac{1}{c_5}\frac{\partial}{\partial\tau}\frac{1}{c}\frac{\partial a^5}{\partial t}=\frac{1}{c}\frac{\partial}{\partial t}\left(\frac{1}{c_5}\frac{\partial a^5}{\partial\tau}\right)=-\frac{1}{c}\frac{\partial C}{\partial t} \tag{19}
$$

and similarly

$$
\boldsymbol{\epsilon} = \sigma\frac{1}{c_5}\frac{\partial\mathbf{a}}{\partial\tau}-\nabla a^5\ \rightarrow\ -\nabla a^5 \tag{20}
$$

$$
\frac{1}{c_5}\frac{\partial\boldsymbol{\epsilon}}{\partial\tau} = -\frac{1}{c_5}\frac{\partial}{\partial\tau}\nabla a^5=-\nabla\left(\frac{1}{c_5}\frac{\partial a^5}{\partial\tau}\right)=\nabla C. \tag{21}
$$

The four homogeneous pre-Maxwell equations now take the form

$$
\nabla\times\mathbf{e}+\frac{1}{c}\frac{\partial\mathbf{b}}{\partial t} = 0\ \rightarrow\nabla\times\mathbf{E}+\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}=0 \tag{22}
$$

$$
\nabla\cdot\mathbf{b} = 0\rightarrow\nabla\cdot\mathbf{B}=0 \tag{23}
$$

and

$$\nabla \times \boldsymbol{\epsilon} - \sigma \frac{1}{c_5} \frac{\partial \mathbf{b}}{\partial \tau} = 0 \rightarrow \nabla \times \boldsymbol{\epsilon} = -\nabla \times \nabla a^5 \equiv 0 \tag{24}$$

$$\nabla \epsilon^0 + \frac{1}{c} \frac{\partial \boldsymbol{\epsilon}}{\partial t} + \sigma \frac{1}{c_5} \frac{\partial \mathbf{e}}{\partial \tau} = 0 \rightarrow \nabla \left(\frac{1}{c} \frac{\partial a^5}{\partial t} \right) + \frac{1}{c} \frac{\partial}{\partial t} \left(-\nabla a^5 \right) \equiv 0. \tag{25}$$

The first inhomogeneous pre-Maxwell equation becomes

$$\nabla \cdot \mathbf{e} - \frac{1}{c_5} \frac{\partial \epsilon^0}{\partial \tau} = e j^0 \qquad \longrightarrow \qquad \nabla \cdot \mathbf{E} + \frac{1}{c} \frac{\partial C}{\partial t} = e j^0$$

reproducing the Gauss law in EED. The second inhomogeneous pre-Maxwell equation becomes

$$\nabla \times \mathbf{b} - \frac{1}{c} \frac{\partial \mathbf{e}}{\partial t} - \frac{1}{c_5} \frac{\partial \boldsymbol{\epsilon}}{\partial \tau} = e \mathbf{j} \qquad \longrightarrow \qquad \nabla \times \mathbf{B} - \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} - \nabla C = e \mathbf{j}$$

reproducing the corresponding expression in EED. The third inhomogeneous pre-Maxwell equation becomes a wave equation for the fifth potential

$$\nabla \cdot \boldsymbol{\epsilon} + \frac{1}{c} \frac{\partial}{\partial t} \epsilon^0 = -\nabla^2 a^5 + \frac{1}{c^2} \frac{\partial^2 a^5}{\partial t^2} = e j^5 \left(x, \tau \right) \tag{26}$$

which can be solved as a Liénard-Wiechert potential from the current j^5 as

$$a^5 \left(x, \tau \right) = \frac{1}{4\pi} \int \frac{d^3 x'}{|\mathbf{x} - \mathbf{x}'|} j^5 \left(t - \frac{|\mathbf{x} - \mathbf{x}'|}{c}, \mathbf{x}', \tau \right) \tag{27}$$

and

$$C = -\frac{\partial a^5 \left(x, \tau \right)}{\partial \tau} \tag{28}$$

determines C. Alternatively, one can specify the field C and interpret the RHS of (26) as an effective current.

In summary, the EED field structure can be described as the most general configuration of SHP fields in which the **E** and **B** fields are τ-independent and behave like Maxwell fields. This is, of course, a very important case in electromagnetism.